化学工学技士(基礎)2018日解答

問1

1) $dQ = u(2\pi r)dr = u_{max} \times \left\{1-\left(\frac{r}{R}\right)^2\right\} 2\pi r\, dr$

$Q = u_{max} \times 2\pi \left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R$

$= u_{max} \times 2\pi \times \frac{R^2}{4} = \underline{\frac{\pi R^2}{2} u_{max}}$ (1)

$(1-A)^3$

$\bar{u} = \frac{Q}{A} = \frac{Q}{\pi R^2} = \frac{1}{2} u_{max}$　$u_{max} = \underline{2\bar{u}}$ (14)

$= 1^3 + 3A^2 - 3A - A^3$

2) $E_K = \int_0^R \frac{1}{2}\rho u^3 2\pi r dr = \int_0^R \frac{1}{2}\rho \cdot \left\{1-\left(\frac{r}{R}\right)^2\right\}^3 2\pi r\, dr \times u_{max}$

$\frac{1}{4} - \frac{1}{8} = \frac{1}{8}$

$= u_{max}^3 \rho\pi \int_0^R \left\{r - 3\cdot\left(\frac{r}{R}\right)^2 \cdot r + 3\left(\frac{r}{R}\right)^4 \cdot r - \left(\frac{r}{R}\right)^6 \cdot r\right\} dr$

$= u_{max}^3 \rho\pi \left(\frac{1}{2}R^2 - \frac{3}{4}R^2 + \frac{1}{2}R^2 - \frac{1}{8}R^2\right) = \rho\pi \cdot \frac{1}{8} R^2 u_{max}^3 = \frac{1}{4}\rho Q u_{max}^2 = \rho Q \bar{u}^2$

$e_K = \frac{E_K}{\rho Q} = \frac{\alpha}{2}\bar{u}^2 = \bar{u}^2$　$\underline{\alpha = 2}$ (14)

$\frac{u}{u_{max}} = \left(\frac{y}{R}\right)^{1/7}$

$Q = u_{max} \times \int_R^0 \left(\frac{y}{R}\right)^{1/7} \times 2\pi(R-y)\ (-dy)$

$= 2\pi u_{max} \cdot \int_0^R \left(\frac{y}{R}\right)^{1/7} \cdot (R-y)\ dy$

$= 2\pi u_{max} \left[\frac{7}{8} \times R^{6/7} \cdot y^{8/7} - R^{-1/7} \cdot y^{15/7} \times \frac{7}{15}\right]_0^R$

$= 2\pi u_{max} \left(\frac{7}{8} \times R^2 - \frac{7}{15} \times R^2\right) = 2\pi u_{max} \times \frac{49}{120} R^2 = \frac{49}{60}\pi R^2 u_{max}$

$\bar{u} = \frac{Q}{A} = \frac{49}{60} u_{max}$.　$u_{max} = \frac{60}{49}\bar{u} = \underline{1.22\,\bar{u}}$ (10)