A5. 非定常熱収支式は

$$wC_p\frac{dT}{d\theta} = q - UA(T-T_m)$$

1) 断熱より. $UA(T-T_m)=0$.

よって $wC_p\frac{dT}{d\theta} = q$

$$wC_p\Delta T = q\Delta\theta$$

$$\Delta T = \frac{q\Delta\theta}{wC_p} = \frac{2.1\times60\times60}{1000\times0.04\times4.2} = 45$$

$$T_2 = T_1+\Delta T = 45+20 = 65\,^\circ C$$

2) 1.5℃分のずれがあり. それが逃げた熱量なので

$$Q_{out} = wC_p\times1.5 = 1000\times0.04\times4.2\times1.5 = 252\ kJ$$

3) $q=0.16$, 「十分長い時間後、水温は70℃で変化しなくなった」$\rightarrow \frac{dT}{d\theta}=0$.

よって $q = UA(T-T_m)$

$$UA = \frac{q}{T-T_m} = \frac{0.16}{70-20} = 0.0032\ kW/K$$

4)

$$\frac{dT}{d\theta} = -\frac{UA}{wC_p}\left(T-T_m-\frac{q}{UA}\right)$$

$\Delta T = T-T_m-\frac{q}{UA}$ とおくと

$$\frac{d\Delta T}{d\theta} = -\frac{UA}{wC_p}\Delta T$$

よって $\Delta T = \alpha e^{\left(-\frac{UA}{wC_p}\theta\right)}$

$\theta=0$ で $T=T_0$ より

$$\Delta T_0 = T_0-T_m-\frac{q}{UA} = \alpha.$$

故に $$T = \left(T_0-T_m-\frac{q}{UA}\right)e^{\left(-\frac{UA}{wC_p}\theta\right)} + T_m + \frac{q}{UA}$$

$$= \left(T_0-\frac{q+UAT_m}{UA}\right)e^{\left(-\frac{UA}{wC_p}\theta\right)} + \frac{q+UAT_m}{UA}$$