2018 - B

B2-1

(a) $r_R = -r_A - r_R = k_1 C_A - k_2 C_R$

$C_R - C_{R0} = C_R = \tau r_R = \tau(-r_A - r_S) = \tau(k_1 C_A - k_2 C_R)$

(b) $C_A - C_{A0} = -\tau k_1 C_A$

$C_A/C_{A0} - 1 = -\tau k_1 C_A/C_{A0}$

よって $C_A/C_{A0} = 1/(1+\tau k_1)$

(c) (a)より $C_R = \dfrac{\tau k_1 C_A}{1+\tau k_2}$

$C_R/C_{A0} = \dfrac{\tau k_1}{1+\tau k_2} \cdot \dfrac{C_A}{C_{A0}} = \dfrac{\tau k_1}{(1+\tau k_1)(1+\tau k_2)} = \dfrac{1}{\left(\frac{1}{\tau k_1}+1\right)(1+\tau k_2)}$

(d) $C_S = (C_{A0} - C_A) - C_R$

$C_S/C_{A0} = 1 - C_A/C_{A0} - C_R/C_{A0} = 1 - \dfrac{1}{1+\tau k_1} - \dfrac{\tau k_1}{(1+\tau k_1)(1+\tau k_2)} = \dfrac{k_1 k_2 \tau^2}{(1+k_1\tau)(1+\tau k_2)}$

(e) $f(\tau) = \dfrac{\tau k_1}{(1+\tau k_1)(1+\tau k_2)}$ とすると

$f'(\tau) = \dfrac{-k_1(k_1 k_2 \tau^2 - 1)}{(1+\tau k_1)^2(1+\tau k_2)^2}$

$\tau = \sqrt{\dfrac{1}{k_1 k_2}}$ で $f(\tau) = 0$ となる。

τ	0	…	τ_{max}	…
$f'(\tau)$	k_1	+	0	−
$f(\tau)$	0	↗	$f(\tau_{max})$	↘

右の増減表より $\tau_{max} = \sqrt{\dfrac{1}{k_1 k_2}}$

(f) $f(\tau_{max}) = \dfrac{1}{\left(\frac{1}{\tau_{max} k_1}+1\right)(1+\tau_{max} k_2)}$

$= \dfrac{1}{\left(1+\sqrt{\frac{k_2}{k_1}}\right)^2}$