A8

(a) $Re_p = \frac{\rho u d_p}{\mu}$

(b) $C_D = 24/Re_p = \frac{24\mu}{\rho u d_p}$ を代入すると

$F = \underline{3\pi\mu u d_p}$

(c) 粘性力

(d) 浮力なので $\underline{V_p \rho g}$

(e) 値を代入して $u_t = \underline{0.15}$ m/s

(f) (2)式より $V_p\rho_p \frac{du}{dt} = V_p g(\rho_p-\rho) - 3\pi\mu u d_p$

$\frac{du}{dt} = \frac{\rho_p-\rho}{\rho_p}g - \frac{3\pi\mu u d_p}{V_p\rho_p}$ ここで、$V_p = \frac{\pi}{6}d_p^3$ より

$= \frac{\rho_p-\rho}{\rho_p}g - \frac{18\mu}{d_p^2\rho_p}u = \frac{u_t}{\tau} - \frac{u}{\tau} = \underline{\frac{1}{\tau}(u_t-u)}$

(g) $u_t - u = y$ とおくと $u = u_t - y$、よって $\frac{du}{dt} = \frac{du}{dy}\cdot\frac{dy}{dt} = -\frac{dy}{dt}$

$-\frac{dy}{dt} = \frac{1}{\tau}\cdot y$

$\frac{dy}{y} = -\frac{dt}{\tau}$

$\int\frac{dy}{y} = -\frac{dt}{\tau}$

$\ln y = -\frac{t}{\tau} + C$

$\ln(u_t-u) = -\frac{t}{\tau} + C$

$t=0$ で $u=0$

$C = \ln u_t$

$\ln\frac{u_t-u}{u_t} = -\frac{t}{\tau}$

よって $u = \underline{u_t(1-e^{-\frac{t}{\tau}})}$

(h) $u_{0.9} = 0.9u_t = u_t(1-e^{-\frac{t}{\tau}})$

これを解くと、$t_{0.9} = 0.0354 \approx \underline{0.036}$

(i) $S_{0.9} = \int_0^{t_{0.9}} u_t(1-e^{-\frac{t}{\tau}})dt$

$= \left[u_t(t+\tau e^{-\frac{t}{\tau}})\right]_0^{t_{0.9}}$

$= \underline{3.3\times10^{-3}}$